Radiative generation of the CPT-even gauge term of the SME from a dimension-five nonminimal coupling term

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In this letter we show for the first time that the usual CPT-even gauge term of the standard model extension (SME) can be radiatively generated, in a gauge invariant level, in the context of a modified QED endowed with a dimension-five nonminimal coupling term recently proposed in the literature. As a consequence, the existing upper bounds on the coefficients of the tensor (K_F) can be used improve the bounds on the magnitude of the nonminimal coupling, $\lambda\,(K_F)$, by the factors 10^5 or 10^{25}. The nonminimal coupling also generates higher-order derivative contributions to the gauge field effective action quadratic terms.

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INTRODUCTION

During the last years it has been a great interest in theories endowed with Lorentz symmetry violation. This interest was initially motivated by the possibility of occurring this kind of violation in high energy theories defined at the Planck energy scale [1]. The Standard Model Extension (SME) [2] is the theoretical effective structure that includes Lorentz-violating (LV) terms, generated as vacuum expectation values of tensors quantities, in the different sectors of the usual Standard Model. A large number of investigations in LV theories have been developed in recent years, addressing distinct sectors of the SME: fermion systems [3], the CPT-odd gauge sector [4–6], the CPT-even gauge sector [7–9]. Interesting theoretical generalizations involving higher dimensional LV operators have also been devised [10–12]. These several studies have served both to elucidate the effects engendered by Lorentz violation and to set up stringent upper bounds on the LV coefficients [13].

Another way to consider Lorentz violation effect in a usual physical theory is by inserting new terms of interaction (LV nonminimal coupling terms) in the Lagrangian. A pioneering study in this sense was undertaken in Ref. [14], in which it was proposed a Lorentz-violating and CPT-odd nonminimal coupling between fermions and the gauge field, $D_\mu = \partial_\mu + ieA_\mu + i\frac{g}{2}\epsilon_{\mu\lambda\alpha\beta}(k_{AF})^\lambda F^{\alpha\beta}$, in the context of the Dirac equation. Here, $(k_{AF})^\mu$ is the Carroll-Field-Jackiw four-vector, and g is the constant that measures the nonminimal coupling magnitude, $[g] = mass^{-1}$. This nonrenormalizable theoretical proposal has been investigated in several distinct tree-level scenarios [15–19]. See also the references inside [19].

Another very interesting question examined in the literature is the radiative generation of LV terms belonging to the SME framework. This topic was first addressed in the end of 90's in Refs. [20], in which it was argued that the CPT-odd or Carroll-Field-Jackiw term, $\epsilon^{\mu\nu\rho\sigma}\left(k_{AF}\right)_\mu A_\nu F_{\rho\sigma}$, belonging to the electrodynamics of the SME is radiatively induced from the axial fermion LV coupling, $b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$. This process leads to the one-loop induced self-energy, $\Pi^{\mu\nu} = \kappa\epsilon^{\mu\nu\alpha\beta}b_\alpha q_\beta$, whose coefficient κ depends explicitly on the regularization prescription used to control the UV divergencies. Such ambiguity has unleashed a great controversy about the possibility the CFJ term to be radiatively generated or not, since a gauge invariant prescription, in principle, would provide $\kappa = 0$ [21]. Other developments in radiative generation, including the induction of the Chern-Simons-like action in a LV massless QED, finite temperature effects, and photon triple splitting, were addressed in Ref. [22].

In Ref. [23] it was demonstrated that aether-like [24], CPT-even, and Lorentz-violating terms can be properly generated when the suitable couplings to the spinor fields are considered. In the first work of Ref. [25] the first higher derivative (dimension-five) CPT-odd operator of the extended QED proposed in Ref. [10] was radiatively induced by the fermion sector term involving the coefficient $g^{\alpha\mu\nu}$. The radiative generation of other higher-dimensional gauge terms, including the Myers-Pospelov one, was achieved in the second work of Ref. [25], starting from a modified QED based on the presence of the CPT-odd nonminimal coupling of Ref. [14]. An interesting study about a QED model also modified by the nonminimal coupling $\epsilon_{\mu\lambda\alpha\beta}\gamma^\mu b^\lambda F^{\alpha\beta}$ was performed in Ref. [26], where it was shown that the one-loop quantum corrections to the photon self-energy could provide two LV terms in the photon sector: the usual CPT-odd Carroll-Field-Jackiw term and the CPT-even aether one, $b_\alpha b_\mu F^{\alpha\beta}F^{\mu\nu}$, first generated in Ref. [23]. This aether term may eventually recover some components of CPT-even usual term, $\left(K_F\right)_{\mu\nu\alpha\beta}$, but not its entire structure. Note that the term $b_\alpha b_\mu$ is just a piece of the representation of the $\left(K_F\right)_{\mu\nu\alpha\beta}$ in terms of the vector b_μ, stated in Eq. (17) of Ref. [26], which maps only the nonbire-

fringent sector of the tensor (K_F). In Ref. [27], a similar investigation was performed considering a chiral version of this nonminimal coupling, that is, $\epsilon_{\mu\lambda\alpha\beta}\gamma_5\gamma^\mu b^\lambda F^{\alpha\beta}$, with analogous results.

In two very recent works [28, 29], it has been proposed a new CPT-even, dimension-five, nonminimal coupling linking the fermionic and gauge fields, in the Dirac equation, $(i\gamma^\mu D_\mu - m)\Psi = 0$, where D_μ is a nonminimal covariant derivative,

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda}{2}\left(K_F\right)_{\mu\nu\alpha\beta}\gamma^\nu F^{\alpha\beta}, \qquad (1)$$

in which $(K_F)_{\mu\nu\alpha\beta}$ is the CPT-even tensor of Abelian gauge sector of the SME, and the nonminimal coupling constant λ has dimension mass^{-1}. The corresponding fermionic Lagrangian density is

$$\mathcal{L}_\Psi = \bar{\Psi}\left[i\partial\!\!\!/ - eA\!\!\!/ - m + \frac{\lambda}{2}\left(K_F\right)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}F^{\alpha\beta}\right]\Psi, \quad (2)$$

with Ψ representing a Dirac usual spinor, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

In Ref. [28], one has studied the effects implied by this new term on the cross section of the electron-muon scattering. The analysis of the ultrarelativistic limit and the available experimental data has allowed to attain the upper bound, $|\lambda\left(K_F\right)| \leq 10^{-12}\left(\mathrm{eV}\right)^{-1}$. On the other hand, the role played by this nonminimal coupling on the nonrelativistic regime of the Dirac equation was analyzed in Ref. [29], focusing on new corrections induced on the hydrogen spectrum and on the gyromagnetic constant. Such analysis has implied an upper bound as restrictive as $|\lambda\left(K_F\right)| \leq 10^{-16}\left(\mathrm{eV}\right)^{-1}$.

In this letter, we show for the first time the radiative generation of the full CPT-even term of the SME electrodynamics, $\left(K_F\right)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$, embracing the entire structure of the tensor such $\left(K_F\right)_{\mu\nu\alpha\beta}$. This is performed by means of a gauge invariant way by starting from the nonminimal CPT-even coupling (1) introduced in the Dirac equation. We finalize presenting the second order contributions in the tensor (K_F) to the photon self-energy. In the conclusions, we discuss how these results may improve some previous upper bounds on the magnitude of the CPT-even nonminimal coupling, yielding $|\lambda_e\left(K_F\right)| < 10^{-21}\left(\mathrm{eV}\right)^{-1}$ and $|\lambda_e\left(K_F\right)| < 10^{-41}\left(\mathrm{eV}\right)^{-1}$ for the nonbirefringent and birefringent coefficients, respectively.

EFFECTIVE ACTION

The QED model under consideration is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_\Psi, \qquad (3)$$

where L_Ψ is given in Eq.(2), and a convenient gauge fixing term must be introduced to properly define the quantization procedure. Firstly, we are interested in the contributions of the fermionic fields, undergone to the nonminimal coupling interaction, to the effective action of the electromagnetic field. The full contribution of the fermion fields to the gauge field effective action is attained by integrating on the fermionic field, yielding

$$e^{iW[A]} = \frac{\det\left(i\partial\!\!\!/ - B - m\right)}{\det\left(i\partial\!\!\!/ - m\right)}, \qquad (4)$$

with the matrix B operator defined as

$$B = eA\!\!\!/ - \lambda\left(K_F\right)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}\partial^\alpha A^\beta. \qquad (5)$$

We are interested in the quadratic term in the gauge field coming from Eq. (4), which is equivalent to the second order contribution in e and λ, written in the momentum space as

$$W^{(2)}\left[A\right] = -\frac{1}{2}\int\frac{d^4q}{\left(2\pi\right)^4}\tilde{A}_\mu\left(-q\right)\Pi^{\mu\nu}\left(q\right)\tilde{A}_\nu\left(q\right), \qquad (6)$$

where $\Pi^{\mu\nu}(q)$ is the one-loop photon self-energy

$$\Pi^{\mu\nu}\left(q\right) = -i\int\frac{d^4p}{\left(2\pi\right)^4}\mathrm{tr}\left[\tilde{S}\left(p\right)V^\mu\tilde{S}\left(p+q\right)V^\nu\right]. \qquad (7)$$

The symbol "tr" denotes the trace operation in the Dirac's indices, $\tilde{S}(p)$ is the fermionic Feynman propagator,

$$\tilde{S}(p) = i\left(p\!\!\!/ - m\right)^{-1}, \qquad (8)$$

and the quantities $\tilde{B}(q)$ and $V_\beta(q)$ are given by

$$\tilde{B}\left(q\right) = V_\beta\left(q\right)\tilde{A}^\beta\left(q\right), \qquad (9)$$

$$V_\beta\left(q\right) = e\gamma_\beta + i\lambda\left(K_F\right)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}q^\alpha. \qquad (10)$$

We can justify the introduction in Eq. (6) of the 1-loop photon self-energy as part of the two-point component of the gauge field effective action because it will allow to show that the radiative corrections preserve the transversality condition, guaranteeing the gauge invariance at this level.

One-loop vacuum polarization

In order to evaluate the one-loop corrections to the photon self-energy, expression (7) is rewritten as the sum

$$\Pi^{\mu\nu}\left(q\right) = \sum_{(a,b)}\Pi^{\mu\nu}_{(a,b)}\left(q\right), \qquad (11)$$

where $\Pi^{\mu\nu}_{(a,b)}(q)$ is defined by

$$\Pi^{\mu\nu}_{(a,b)}\left(q\right) = i\int\frac{d^4p}{\left(2\pi\right)^4}\frac{N^{\mu\nu}_{(a,b)}}{\left(p^2 - m^2\right)\left(\left(p+q\right)^2 - m^2\right)}. \qquad (12)$$

It is obtained by replacing the fermion Feynman propagators (8) in Eq. (7), so that $N^{\mu\nu}_{(a,b)}$ is

$$N^{\mu\nu}_{(a,b)} = \mathrm{tr}\left[(\not{p}+m)V^{\mu}_{(a)}\left(\not{p}+\not{q}+m\right)V^{\nu}_{(b)}\right], \qquad (13)$$

with $a,b=0,1$ representing the usual and modified vertices,

$$V^{\mu}_{(0)} = e\gamma^{\mu}, \quad V^{\mu}_{(1)} = i\lambda\left(K_F\right)^{\alpha\beta\chi\mu}\sigma_{\alpha\beta}q_{\chi}, \qquad (14)$$

respectively. The integral in (12) is a quadratically divergent by power counting requiring some regularization technique, in our case, we will use an explicitly gauge invariant prescription: the dimensional regularization. The dimensional regularization works in $D=4-2\epsilon$ dimensions, regarding $\epsilon\to 0^{+}$.

Hence, in our notation, $\Pi^{\mu\nu}_{(0,0)}$ represents the usual one-loop self-energy corresponding to the vertex $V^{\mu}_{(0)}$ while the new contributions involving the new vertex $V^{\mu}_{(1)}$ namely $\Pi^{\mu\nu}_{(0,1)}, \Pi^{\mu\nu}_{(1,0)}, \Pi^{\mu\nu}_{(1,1)}$.

The first contribution to be considered is the usual one-loop photon self-energy contribution, $\Pi^{\mu\nu}_{(0,0)}$, read as

$$\Pi^{\mu\nu}_{(0,0)}\left(q\right) = ie^{2}\int\frac{d^{4}p}{\left(2\pi\right)^{4}}\frac{N^{\mu\nu}_{(0,0)}}{\left(p^{2}-m^{2}\right)\left(\left(p+q\right)^{2}-m^{2}\right)}, \qquad (15)$$

with

$$N^{\mu\nu}_{(0,0)} = \mathrm{tr}\left[(\not{p}+m)\gamma^{\mu}\left(\not{p}+\not{q}+m\right)\gamma^{\nu}\right]. \qquad (16)$$

By following the dimensional regularization technique, we perform the trace operations and compute the momentum integral with the Feynman parametrization. Next, we retain only the contribution of the divergent terms,

$$\Pi^{\mu\nu}_{(0,0)}(q) = -\frac{1}{12\pi^{2}\epsilon}\left(g^{\mu\nu}q^{2}-q^{\mu}q^{\nu}\right), \qquad (17)$$

for the quadratic term of the gauge field effective action, given as

$$W^{(2)}_{(0,0)}\left[A\right] = -\frac{e^{2}}{48\pi^{2}\epsilon}\int d^{4}x\; F^{\mu\nu}F_{\mu\nu}. \qquad (18)$$

As expected, the usual vertex induces a counterterm proportional to the Maxwell term, $F^{\mu\nu}F_{\mu\nu}$, which is already present in the QED action.

We now go on evaluating the terms $\Pi^{\mu\nu}_{(0,1)}, \Pi^{\mu\nu}_{(1,0)}$. A preliminary analysis allows to notice that $\Pi^{\mu\nu}_{(0,1)} = \Pi^{\mu\nu}_{(1,0)}$, so we go compute the first one, in which the replacement of the vertices (14) yields

$$\Pi^{\mu\nu}_{(0,1)}\left(q\right) = -e\lambda q_{\sigma}\left(K_F\right)_{\alpha\beta}{}^{\sigma\nu}\Pi^{\mu\alpha\beta}, \qquad (19)$$

where

$$\Pi^{\mu\alpha\beta} = \int\frac{d^{4}p}{\left(2\pi\right)^{4}}\frac{N^{\mu\alpha\beta}}{\left(p^{2}-m^{2}\right)\left[(p+q)^{2}-m^{2}\right]}, \qquad (20)$$

and

$$N^{\mu\alpha\beta} = \mathrm{tr}\left[\left(\not{p}+m\right)\gamma^{\mu}\left(\not{p}+\not{q}+m\right)\sigma^{\alpha\beta}\right]. \qquad (21)$$

We calculate $\Pi^{\mu\alpha\beta}$ by following the same procedure used to compute the quantity (15), thus the divergent term is

$$\Pi^{\mu\alpha\beta} = -\frac{m}{4\pi^{2}\epsilon}\left(q^{\alpha}g^{\beta\mu}-q^{\beta}g^{\alpha\mu}\right), \qquad (22)$$

which yields the following contribution to the photon self-energy:

$$\Pi^{\mu\nu}_{(0,1)}\left(q\right) = \frac{me\lambda}{4\pi^{2}\epsilon}\left(K_F\right)_{\alpha\beta}{}^{\sigma\nu}q_{\sigma}\left(q^{\alpha}g^{\beta\mu}-q^{\beta}g^{\alpha\mu}\right). \qquad (23)$$

Inserting it in the effective action, one attains

$$W^{(2)}_{(0,1)}\left[A\right]+W^{(2)}_{(1,0)}\left[A\right] = \frac{me\lambda}{8\pi^{2}\epsilon}\int d^{4}x\;\left(K_F\right)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}. \qquad (24)$$

This result reveals that the CPT-even abelian gauge term of the SME, $\left(K_F\right)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$, is radiatively induced by the new vertex. This is the first time this full CPT-even term is generated by a gauge-invariant mechanism.

We finalize evaluating the term $\Pi^{\mu\nu}_{(1,1)}$, which after vertex substitution can be rewritten as

$$\Pi^{\mu\nu}_{(1,1)}\left(q\right) = -i\lambda^{2}\left(K_F\right)_{\eta\theta}{}^{\xi\mu}\left(K_F\right)_{\lambda\rho}{}^{\chi\nu}q_{\xi}q_{\chi}\Pi^{\eta\theta\lambda\rho}\left(q\right), \qquad (25)$$

where we have defined

$$\Pi^{\eta\theta\lambda\rho}\left(q\right) = \int\frac{d^{4}p}{\left(2\pi\right)^{4}}\frac{N^{\eta\theta\lambda\rho}}{\left(p^{2}-m^{2}\right)\left[(p+q)^{2}-m^{2}\right]}, \qquad (26)$$

and

$$N^{\eta\theta\lambda\rho} = \mathrm{tr}\left[(\not{p}+m)\sigma^{\eta\theta}\left(\not{p}+\not{q}+m\right)\sigma^{\lambda\rho}\right]. \qquad (27)$$

Direct computation of the integral (26) allows to get the following divergent terms

$$\begin{aligned}\Pi^{\eta\theta\lambda\rho}\left(q\right) = &-\frac{i}{4\pi^{2}\epsilon}\left(m^{2}-\frac{q^{2}}{6}\right)\left[g^{\eta\rho}g^{\theta\lambda}-g^{\eta\lambda}g^{\theta\rho}\right] \\ &-\frac{i}{12\pi^{2}\epsilon}\left[q^{\eta}q^{\rho}g^{\theta\lambda}-q^{\eta}q^{\lambda}g^{\theta\rho}+q^{\theta}q^{\lambda}g^{\eta\rho}-q^{\theta}q^{\rho}g^{\eta\lambda}\right],\end{aligned} \qquad (28)$$

providing the second-order LV contributions to the photon self-energy,

$$\begin{aligned}\Pi^{\mu\nu}_{(1,1)}\left(q\right) = &-\frac{m^{2}\lambda^{2}}{2\pi^{2}\epsilon}\left(K_F\right)^{\mu\xi\lambda\rho}\left(K_F\right)_{\lambda\rho}{}^{\chi\nu}q_{\xi}q_{\chi} \\ &+\frac{\lambda^{2}}{12\pi^{2}\epsilon}\left(K_F\right)^{\mu\xi\lambda\rho}\left(K_F\right)_{\lambda\rho}{}^{\chi\nu}q^{2}q_{\xi}q_{\chi} \\ &+\frac{\lambda^{2}}{3\pi^{2}\epsilon}\left(K_F\right)^{\mu\xi\eta\lambda}\left(K_F\right)_{\lambda}{}^{\rho\chi\nu}q_{\xi}q_{\eta}q_{\rho}q_{\chi},\end{aligned} \qquad (29)$$

and the following counterterms to the effective gauge field action,

$$W_{(1,1)}^{(2)} = -\frac{\lambda^2 m^2}{16\pi^2\epsilon}\int d^4x\ (K_F)^{\mu\xi\rho\theta}(K_F)_{\rho\theta}{}^{\chi\nu}F_{\mu\xi}F_{\chi\nu} + \frac{\lambda^2}{96\pi^2\epsilon}\int d^4x\ (K_F)^{\mu\xi\rho\theta}(K_F)_{\rho\theta}{}^{\chi\nu}F_{\mu\xi}\Box F_{\chi\nu} - \frac{\lambda^2}{24\pi^2\epsilon}\int d^4x\ (K_F)^{\mu\xi\eta\theta}(K_F)_{\theta}{}^{\lambda\chi\nu}F_{\mu\xi}(\partial_\eta\partial_\lambda)F_{\chi\nu}. \quad (30)$$

We can note that the first term is a dimension-four operator while the two last are dimension-six operators.

Finally, we can show starting from Eqs. (17,23,29) that the divergent contributions to the vacuum polarization tensor are purely transversal. Thus, a direct verification yields,

$$q_\nu \Pi^{\mu\nu}(q) = 0, \quad (31)$$

It assures, at 1-loop level, the absence of gauge anomalies and, consequently, gauge symmetry preservation in the context of the modified QED of Lagrangian (2).

CONCLUSIONS

In this work, we have studied the contributions to the effective action of the electromagnetic field induced by the dimension-five nonminimal coupling, $\lambda(K_F)_{\mu\nu\alpha\beta}F^{\alpha\beta}\bar{\psi}\sigma^{\mu\nu}\psi$. Specifically, we have focused in the quadratic gauge field terms generated by the 1-loop radiative corrections. Our main result is that such contributions have generated the CPT-even term of the SME electrodynamics, $(K_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$. Furthermore, at second order in (K_F), CPT-even terms containing fourth-order derivatives involving dimension-six operators were also produced.

Our analysis has shown that the first-order LV correction to the one-loop vacuum polarization leads to the dimension-four CPT-even term of the SME, $\left(me\lambda/8\pi^2\right)(K_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, as presented in Eq. (24). This implies that the Maxwell electrodynamics must be modified by the inclusion of such term in its structure, implying a new LV electrodynamics ruled by the Lagrangian density $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}g\lambda(K_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, where $g = me/2\pi^2$. As a consequence, we can use the same phenomenology that allows to constrain the coefficients of the tensor (K_F) with stringent upper bounds [7–9, 13] to improve the bounds on the magnitude of the quantity $\lambda(K_F)_{\mu\nu\alpha\beta}$ by the factor $1/g \sim 4\times10^{-4}$ (in the electron case). This means that a known upper-bound for the nonbirefringent components, $|K_F| < 10^{-17}$, would lead to an upper-bound as tight as $|\lambda_e(K_F)| < 10^{-21}\,(\mathrm{eV})^{-1}$ for the corresponding nonminimal coupling to the electron-photon interaction. A similar argument can be used to transfer the existing bounds of the birefringent components, $|K_F| < 10^{-37}$, to level of $|\lambda_e(K_F)| < 10^{-41}\,(\mathrm{eV})^{-1}$ while associated with the same electron-photon nonminimal interaction.

Therefore, this analysis allows to improve the previous upper bounds on $|\lambda_e K_F|$ attained in Ref. [29], by the factors 10^5 and 10^{25}, concerning to the nonbirefringent and birefringent sectors of the nonminimal coupling, respectively. As for the higher-order derivative terms in Eq. (30), radiatively generated at order λ^2, they do not lead to any improvement of the previous upper-bounds. An important fact must be noted, since the counterterm depends on the particle mass, it is reasonable to suppose that the magnitude of the nonminimal coupling may also depend on the particle mass under analysis, fact also remarked in Ref. [29], in which different upper bounds were stated for the electron and proton nonminimal interactions with the electromagnetic field. In respect to the proton nonminimal interaction, previously discussed in Ref. [29], the bounds could now be improved to the level $|\lambda_p(K_F)| < 10^{-25}\,(\mathrm{eV})^{-1}$ and $|\lambda_p K_F| < 10^{-44}\,(\mathrm{eV})^{-1}$, for the nonbirefringent and birefringent components, respectively.

Some additional sensitive issues still remain under investigation. The new contributions implied by this nonminimal interaction to the fermionic sector and its dispersion relations, and the vertex corrections which possibly give contributions to the anomalous magnetic moment, are two interesting research goals. Obviously such corrections are relevant for establishing the 1-loop renormalization of this model. These investigations are now underway.

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